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BRIEF COMMUNICATION

AN APPROXIMATE METHOD FOR ANALYZING NONEQUILIBRIUM ACOUSTIC PHENOMENA IN ATMOSPHERIC FOG

P. CHANDRAN†
Department of Mathematics, Indian Institute of Technology, Delhi, Hauz Khas, New Delhi-110016, India

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The dynamics of a foggy medium is characterized by the presence of different phases of matter in the same flow field necessitating the consideration of additional nonequilibrium processes as compared to conventional gasdynamics. As can be seen from the works of Marble (1969, 1970) and others (see, e.g. Pai 1971; Pai & Hsieh 1973; Soo 1976; Sha & Soo 1978), the main difficulties encountered in analyzing the nonequilibrium features of such a medium have much to do with the source terms corresponding to the gas–droplet interactions as also with the increased number of conservation equations. Acoustic waves propagating in a gas-particle medium containing volatile or nonvolatile particles have been discussed before (Marble 1969, 1970; Marble & Wooten 1970; Marble & Candel 1975; Lyman & Chen 1978; Bhutani & Chandran 1977; Davidson 1977) and their practical applications in physical and industrial fields have been indicated. These studies have brought out clearly the added effect of particulate phase as far as sound dispersion and attenuation are concerned.

In furtherance of the acoustic analysis, the present study has been taken up with a view to fruitfully apply Whitham's (1959) approximation technique of reducing higher order wave equations to lower order equations as applied to a foggy medium. As has been demonstrated before by Cogley & Vincenti (1969), such an approach has been seen to yield additional insights into the propagation features of small disturbances involving, among others, the predominance of specific waveforms in specific temporal regions.

THE ACOUSTIC WAVE EQUATIONS

When the dynamical equations of a gas–particle system undergoing phase change are subjected to linear acoustic approximations, a single wave equation satisfying any field variable ϕ has been derived by Marble (1969). This equation can be recast into the dimensionless form

$$\left[\frac{\partial^3}{\partial t^3}\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}\right)+\alpha_1\frac{\partial^2}{\partial t^2}\left(\frac{1}{a_1^2}\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}\right)+\alpha_2\frac{\partial}{\partial t}\left(\frac{1}{a_2^2}\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}\right)+\alpha_3\left(\frac{1}{a_3^2}\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}\right)\right]\phi=0, \quad [1]$$

where

$$\alpha_1^2=\alpha_v+\alpha_T+\alpha_D,\ \ \alpha_2=\alpha_v\alpha_T+\alpha_v\alpha_D+\alpha\alpha_T\alpha_D,\ \ \alpha_3=\alpha\alpha_v\alpha_T\alpha_D,$$

$$a_1^2=\alpha_1\left(\frac{\alpha_v}{a_v^2}+\frac{\alpha_T}{a_T^2}+\frac{\alpha_D}{a_D^2}\right)^{-1},$$

$$a_2^2=\alpha_2\left(\frac{\alpha_v\alpha_T}{a_{vT}^2}+\frac{\alpha_v\alpha_D}{a_{vD}^2}+\frac{\alpha\alpha_T\alpha_D}{a_{TD}^2}\right)^{-1},$$

$$\alpha_v=(\omega\tau_v)^{-1},\ \ \alpha_T=\{(c_p/c)+k_P\}(\omega\tau_T)^{-1},$$

$$\alpha_D=\frac{[\gamma/(\gamma-1)](c_P/c)k_v\eta^2+k_P(1-k_v)}{\omega\tau_D},$$

†Present address: Department of Engineering Mechanics, 155, West Woodruff Avenue, The Ohio State University, Columbus, OH 43210, U.S.A.

$$\alpha = \frac{k_P}{(c_P/c)+k_P}\left[1+\frac{(c_P/c)(1-k_v)}{[\gamma/(\gamma-1)](c_P/c)k_v\eta^2+k_P(1-k_v)}\right],$$

$$a_v^2 = 1/(1+k_P),\quad a_T^2 = [(c_P/c)+k_P]/[(c_P/c)+\gamma k_P],$$

$$a_D^2 = \frac{\left(\frac{\gamma}{\gamma-1}\right)(c_P/c)k_v\eta^2+k_P(1-k_v)}{\left(\frac{\gamma}{\gamma-1}\right)(c_P/c)k_v\eta^2+k_P(1-k_v+\gamma k_v)},$$

$$a_{vT}^2 = a_T^2/(1+k_P),\quad a_{vD}^2 = a_D^2/(1+k_P),$$

$$a_{TD}^2 = \frac{(1-k_v)\{(c_P/c)+k_P\}+\left(\frac{\gamma}{\gamma-1}\right)(c_P/c)k_v\eta^2}{(1-k_v+\gamma k_v)(c_P/c)+\gamma k_P+(c_P/c)\gamma k_v\eta\left\{\left(\frac{\gamma}{\gamma-1}\right)\eta-2\right\}},$$

$$a_3^2 = a_{TD}^2/(1+k_P),\quad k_P = \rho_P/\rho,\quad k_v = \rho_v/\rho,\quad \eta = h_l/(c_PT_0).$$

In the above, τ_v, τ_T, τ_D are, respectively, the velocity, thermal and diffusion equilibration times, ω the frequency used for nondimensionalizing these, c the specific heat of particle cloud, c_P the specific heat of gas at constant pressure, h_l the latent heat of vaporization and ρ, ρ_P, ρ_v are the densities of gas, particle and vapor phases, respectively. It may also be noted here that the "constrained" sound speeds a_v through a_3 have been nondimensionalized with respect to the frozen speed of sound and the "composite" speeds a_1 and a_2 have been introduced in view of the instabilities of intermediate waves. (For a detailed discussion of the physical features of propagation, see Marble (1969), Bhutani & Chandran (1977)). Also, the dimensionless parameters $1/\alpha_v$, $1/\alpha_T$, $1/\alpha_D$, known as Damköhler numbers, compare typical relaxation parameters τ_v, τ_T, τ_D with a typical flow time $1/\omega$.

In the discussion that follows, it is assumed that the disturbances are right-running waves starting from the origin and spanning the right-half space with assumed homogeneous initial conditions. To get the equivalent lower order equations, we follow first the frozen speed of sound and put $(\partial/\partial t) \simeq -(\partial/\partial x)$ in [1] to yield

$$(\phi_t+\phi_x)_{tt}+\tfrac{1}{2}(\alpha_1\beta_{10}\phi_{tt}+\alpha_2\beta_{20}\phi_t+\alpha_3\beta_{30}\phi)=0, \qquad [2]$$

where

$$\beta_{i0} = \frac{1}{a_i^2}-1.$$

Similarly, following the other speeds a_1, a_2, a_3, [1] reduces, respectively, to

$$\phi_{ttt}-\frac{2\alpha_1}{a_1\beta_{10}}\left(\frac{1}{a_1}\phi_t+\phi_x\right)_t-\frac{1}{\beta_{10}}(\alpha_2\beta_{21}\phi_t+\alpha_3\beta_{31}\phi)=0, \qquad [3]$$

$$\phi_{ttt}-\frac{1}{\beta_{20}}(\alpha_1\beta_{12}\phi_{tt}+\alpha_3\beta_{32}\phi)-\frac{2\alpha_2}{a_2\beta_{20}}\left(\frac{1}{a_2}\phi_t+\phi_x\right)=0, \qquad [4]$$

$$\phi_{tttt}-\frac{1}{\beta_{30}}(\alpha_1\beta_{13}\phi_{ttt}+\alpha_2\beta_{23}\phi_{tt})-\frac{2\alpha_3}{a_3\beta_{30}}\left(\frac{1}{a_3}\phi_t+\phi_x\right)=0, \qquad [5]$$

where

$$\beta_{ij} = \frac{1}{a_i^2}-\frac{1}{a_j^2},\quad (i\neq j>0).$$

Finally, the "degenerate" speed 0 is followed to yield

$$\phi_{ttt} + \alpha_1\phi_{tt} + \alpha_2\phi_t + \alpha_3\phi = 0. \qquad [6]$$

It can be shown that [6] does not in any way help the present study because it requires an inhomogeneous initial condition to represent a unique physical situation. Thus the single equation [1] has been reduced to the system of four lower order equations [2]–[5] each of which contains the relevant signalling operator corresponding to the speed being followed.

ANALYSIS OF HARMONIC DISTURBANCES

We shall use here the system of equations obtained before to study the disturbances generated by the harmonic oscillations of a planar wall. Writing

$$\phi(x,t) = f(x)\,e^{it}$$

we see that the solutions of [2]–[5] can be expressed, respectively, as

$$\phi_k(x,t) = C_k \exp\left[-\delta_k x + i(t - \Lambda_k x)\right],\ (k = 0, 1, 2, 3) \qquad [7]$$

where the dimensionless damping coefficients δ_k and wave speeds $(1/\Lambda_k)$ are given by

$$\delta_0 = \frac{1}{2}(\alpha_1\beta_{10} - \alpha_3\beta_{30}),\ \frac{1}{\Lambda_0} = \left[1 - \frac{\alpha_2\beta_{20}}{2}\right]^{-1},$$

$$\delta_1 = \frac{a_1}{2\alpha_1}(\beta_{10} + \alpha_2\beta_{21}),\ \frac{1}{\Lambda_1} = \frac{2\alpha_1 a_1}{2\alpha_1 - \alpha_3 a_1^2\beta_{31}},$$

$$\delta_2 = \frac{a_2}{2\alpha_2}(\alpha_1\beta_{21} + \alpha_3\beta_{32}),\ \frac{1}{\Lambda_2} = \frac{2\alpha_2 a_2}{2\alpha_2 + a_2^2\beta_{20}},$$

$$\delta_3 = -\frac{a_3}{2\alpha_3}(\beta_{30} + \alpha_2\beta_{23}),\ \frac{1}{\Lambda_3} = \frac{2\alpha_3 a_3}{2\alpha_3 - \alpha_1 a_3^2\beta_{13}}. \qquad [8]$$

As per the approximations involved in the present method, the four solutions, [7], are expected to span the whole region. To see how this is so consider the exact solution

$$\phi(x,t) = C\exp\left[-\delta x + i(t - \Lambda x)\right], \qquad [9]$$

where

$$\left.\begin{matrix}\delta\\ \Lambda\end{matrix}\right\} = \left[\frac{\mp\left\{(\alpha_1-\alpha_3)\left(\frac{\alpha_1}{a_1^2}-\frac{\alpha_3}{a_3^2}\right)+(1-\alpha_2)\left(1-\frac{\alpha_2}{a_2^2}\right)\right\} + \sqrt{\left\{(\alpha_1-\alpha_3)\left(\frac{\alpha_1}{a_1^2}-\frac{\alpha_3}{a_3^2}\right)+(1-\alpha_2)\left(1-\frac{\alpha_2}{a_2^2}\right)\right\}^2 + \left\{(1-\alpha_2)\left(\frac{\alpha_1}{a_1^2}-\frac{\alpha_3}{a_3^2}\right)-(\alpha_1-\alpha_3)\left(1-\frac{\alpha_2}{a_2^2}\right)\right\}^2}}{(\alpha_1-\alpha_3)^2+(1-\alpha_2)^2}\right]^{\frac{1}{2}} \qquad [10]$$

where the upper sign goes with δ and the lower sign with Λ.

The complexity of the result [10] testifies to the desirability of approximate methods. Moreover, this result does not give any idea of the region where each wave form is

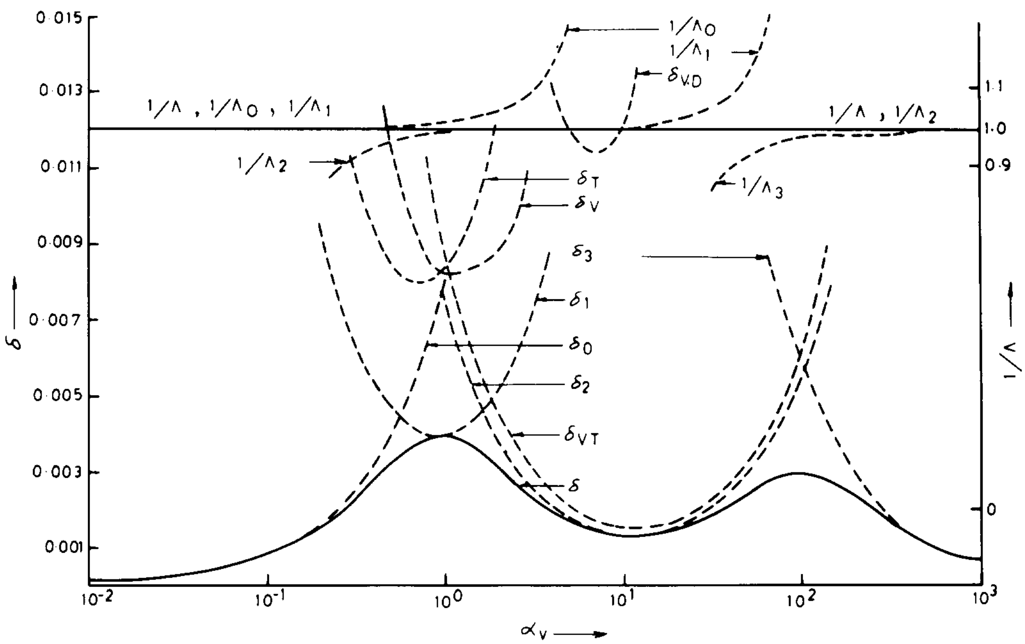

Figure 1. Damping factors and wave speeds. $h_l/(RT_0) = 5.0$, $\gamma = 1.6$, $k_P = k_v = 0.01$. Exact: —, Approximate: ---.

predominant. The expressions of damping factors and wave speeds are functions of η, k_P, k_v, γ, τ_v, τ_T, τ_D and, therefore, a quantitative comparison of the exact and approximate results is still difficult if the number of parameters is not reduced. To this end, we observe that τ_v and τ_T are equal for Prandtl number (2/3) and since the Schmidt number is nearly unity, τ_D is also nearly equal to τ_v and τ_T. Accordingly, we assume that $\tau_v = \tau_T = \tau_D$.

Since the Damköhler number involves both ω and τ_v, before discussing the general wave structure it will be revealing to derive the limiting values of δ and $(1/\Lambda)$ corresponding to the extreme limits of ω and τ_v. As expected, we see that these limiting cases give only the initial and final motion. In order to get a detailed picture of the structure of waves and to see how the four solutions make up the total solution and their regions of validity, we have plotted in figure 1 the approximate and exact damping and wave speeds. It is seen that the approximate results are not uniformly valid as the Damköhler number varies from ∞ to 0. There are transition regions where one solution takes over from the other discontinuously. For instance, in figure 1, δ_0 and $(1/\Lambda_0)$ are valid in the initial stages of high frequency motion till about $\alpha_v = 0.3$ and the solution corresponding to a_1 takes over from the preceding one discontinuously. This is valid in the neighbourhood of $\alpha_v = 1.0$ whereas the region $1 < \alpha_v < 100$ is covered by the solution following the speed a_2. For $\alpha_v \gg 1$, δ_3 intersects δ_2 near $\alpha_v = 100$ and the final motion is given by the equilibrium speed solution. Also, it has been seen that a steady increase in the particle loading results in a gradual decrease in the regions of validity of intermediate waves.

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